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Dimensionless physical constant

In physics, a **dimensionless physical constant**, sometimes called a **fundamental physical constant**, is a physical constant that is dimensionless. It has no units attached and has a numerical value that is independent of the system of units used. Perhaps the best-known example is the fine-structure constant, α, which has an approximate value of $^1/_{137.036}$.

The term *fundamental physical constant* is often used to refer to the dimensionless constants, but has also been used (primarily by NIST[1] and CODATA[2]) to refer to certain universal dimensioned physical constants, such as the speed of light *c*, vacuum permittivity ε_0, Planck constant *h*, and the gravitational constant *G,* that appear in the most basic theories of physics.[1][3][4][5] Other physicists do not recognize this usage, and reserve the use of the term *fundamental physical constant* solely for dimensionless physical constants that cannot be derived from any other source.[6][7] This narrower usage will be followed here.

Contents

Characteristics

There is no exhaustive list of such constants. But it is meaningful to ask about the minimal number of fundamental constants necessary to determine a given physical theory. Thus, the Standard Model requires 25 physical constants, about half of them the masses of fundamental particles (which become "dimensionless" when expressed relative to the Planck mass or, alternatively, relative to the electron mass along with the gravitational coupling constant).

Fundamental physical constants cannot be derived but have to be measured. Development in physics may lead to either a reduction or an extension of their number: discovery of new particles, or new relationships between physical phenomena, would introduce new constants, while on the other hand, the development of a more fundamental theory might allow the derivation of several constants from a more fundamental constant.

A long-sought goal of theoretical physics is to find first principles ("Theory of Everything") from which all of the fundamental dimensionless constants can be calculated and compared to the measured values.

The large number of fundamental constants required in the Standard Model has been regarded as unsatisfactory since the theory's formulation in the 1970s. The desire for a theory that would allow the calculation of particle masses is a core motivation for the search for "Physics beyond the Standard Model".

History

In the 1920s and 1930s, Arthur Eddington embarked upon extensive mathematical investigation into the relations between the fundamental quantities in basic physical theories, later used as part of his effort to construct an overarching theory unifying quantum mechanics and cosmological physics. For example, he speculated on the potential consequences of the ratio of the electron radius to its mass. Most notably, in a 1929 paper he set out an argument based on the Pauli exclusion principle and the Dirac equation that fixed the value of the reciprocal of the fine-structure constant as $\alpha^{-1} = 16 + \frac{1}{2} \times 16 \times (16 - 1) = \mathbf{136}$. When its value was discovered to be closer to 137, he changed his argument to match that value. His ideas were not widely accepted, and subsequent experiments have shown that they were wrong (for example, none of the measurements of the fine-structure constant suggest an integer value; in 2018 it was measured at α = 1/137.035999046(27)).[8]

Though his derivations and equations were unfounded, Eddington was the first physicist to recognize the significance of universal dimensionless constants, now considered among the most critical components of major physical theories such as the Standard Model and ΛCDM cosmology. He was also the first to argue for the importance of the cosmological constant Λ itself, considering it vital for explaining the expansion of the universe, at a time when most physicists (including its discoverer, Albert Einstein) considered it an outright mistake or mathematical artifact and assumed a value of zero: this at least proved prescient, and a significant positive Λ features prominently in ΛCDM.

Eddington may have been the first to attempt in vain to derive the basic dimensionless constants from fundamental theories and equations, but he was certainly not the last. Many others would subsequently undertake similar endeavors, and efforts occasionally continue even today. None have yet produced convincing results or gained wide acceptance among theoretical physicists.[9][10][11]

The mathematician Simon Plouffe has made an extensive search of computer databases of mathematical formulae, seeking formulae for the mass ratios of the fundamental particles.

An empirical relation between the masses of the electron, muon and tau has been discovered by physicist Yoshio Koide, but this formula remains unexplained.

Examples

Dimensionless fundamental physical constants include:

- α, the fine structure constant, the coupling constant for the electromagnetic interaction ($\approx \frac{1}{137}$). Also the square of the electron charge, expressed in Planck units, which defines the scale of charge of elementary particles with charge.
- μ or β, the proton-to-electron mass ratio, the rest mass of the proton divided by that of the electron (≈1836). More generally, the ratio of the rest masses of any pair of elementary particles.
- α_s, the coupling constant for the strong force (≈1)
- α_G, the gravitational coupling constant ($\approx 10^{-45}$) which is the square of the electron mass, expressed in Planck units. This defines the scale of the masses of elementary particles and the ratio of α_G to the other coupling constants has

also been used to express the strength of gravitation relative to the other interactions.

Fine structure constant

One of the dimensionless fundamental constants is the fine structure constant:

$$\alpha = \frac{e^2}{\hbar c \; 4\pi\varepsilon_0} \approx \frac{1}{137.03599908},$$

where e is the elementary charge, $\hbar$ is the reduced Planck's constant, c is the speed of light in a vacuum, and ε_0 is the permittivity of free space. The fine structure constant is fixed to the strength of the electromagnetic force. At low energies, $\alpha \approx 1/137$, whereas at the scale of the Z boson, about 90 GeV, one measures $\alpha \approx 1/127$. There is no accepted theory explaining the value of α; Richard Feynman elaborates:

> There is a most profound and beautiful question associated with the observed coupling constant, *e* – the amplitude for a real electron to emit or absorb a real photon. It is a simple number that has been experimentally determined to be close to 0.08542455. (My physicist friends won't recognize this number, because they like to remember it as the inverse of its square: about 137.03597 with about an uncertainty of about 2 in the last decimal place. It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it.) Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!
>
> — Richard Feynman, Richard P. Feynman (1985). *QED: The Strange Theory of Light and Matter*. Princeton University Press. p. 129. ISBN 0-691-08388-6.

The analog of the fine structure constant for gravitation is the gravitational coupling constant. This constant requires the arbitrary choice of a pair of objects having mass. The electron and proton are natural choices because they are stable, and their properties are well measured and well understood. If α_G is calculated from the masses of two protons, its value is $\approx 10^{-38}$.

Standard model

The original standard model of particle physics from the 1970s contained 19 fundamental dimensionless constants describing the masses of the particles and the strengths of the electroweak and strong forces. In the 1990s, neutrinos were discovered to have nonzero mass, and a quantity called the vacuum angle was found to be indistinguishable from zero.

The complete standard model requires 25 fundamental dimensionless constants (Baez, 2011 (http://math.ucr.edu/home/baez/constants.html)). At present, their numerical values are not understood in terms of any widely accepted theory and are determined only from measurement. These 25 constants are:

- the fine structure constant;

- the strong coupling constant;
- fifteen masses of the fundamental particles (relative to the Planck mass m_P = 1.220 89(6) × 10^{19} GeV/c^2), namely:
  - six quarks
  - six leptons
  - the Higgs boson
  - the W boson
  - the Z boson
- four parameters of the CKM matrix, describing how quarks oscillate between different forms;
- four parameters of the Pontecorvo–Maki–Nakagawa–Sakata matrix, which does the same thing for neutrinos.

Dimensionless constants of the Standard Model			
Symbol	Description	Dimensionless value	Alternative value representation
m_u / m_P	up quark mass	$1.4 \times 10^{-22} – 2.7 \times 10^{-22}$	1.7 – 3.3 MeV/c^2
m_d / m_P	down quark mass	$3.4 \times 10^{-22} – 4.8 \times 10^{-22}$	4.1 – 5.8 MeV/c^2
m_c / m_P	charm quark mass	1.04×10^{-19}	1.27 GeV/c^2
m_s / m_P	strange quark mass	8.27×10^{-21}	101 MeV/c^2
m_t / m_P	top quark mass	1.41×10^{-17}	172.0 GeV/c^2
m_b / m_P	bottom quark mass	3.43×10^{-19}	4.19 GeV/c^2
$\theta_{12,CKM}$	CKM 12-mixing angle	0.23	13.1°
$\theta_{23,CKM}$	CKM 23-mixing angle	0.042	2.4°
$\theta_{13,CKM}$	CKM 13-mixing angle	0.0035	0.2°
δ_{CKM}	CKM CP-violating Phase	0.995	57°
m_e / m_P	electron mass	4.18546×10^{-23}	511 keV/c^2
m_{ν_e} / m_P	electron neutrino mass	below 1.6×10^{-28}	below 2 eV/c^2
m_μ / m_P	Muon mass	8.65418×10^{-21}	105.7 MeV/c^2
m_{ν_μ} / m_P	muon neutrino mass	below 1.6×10^{-28}	below 2 eV/c^2
m_τ / m_P	tau mass	1.45535×10^{-19}	1.78 GeV/c^2
m_{ν_τ} / m_P	tau neutrino mass	below 1.6×10^{-28}	below 2 eV/c^2
$\theta_{12,PMNS}$	PMNS 12-mixing angle	0.5973 ±0.0175	34.22° ±1°
$\theta_{23,PMNS}$	PMNS 23-mixing angle	0.785 ±0.12	45° ±7.1°
$\theta_{13,PMNS}$	PMNS 13-mixing angle	≈0.077	≈4.4°
δ_{PMNS}	PMNS CP-violating Phase	Unknown	
α	fine-structure constant	0.00729735	1/137.036
α_s	strong coupling constant	≈1	≈1
$m_{W^\pm} / m_P$	W boson mass	$(6.5841 \pm 0.0012) \times 10^{-18}$	(80.385 ± 0.015) GeV/c^2
m_{Z^0} / m_P	Z boson mass	$(7.46888 \pm 0.00016) \times 10^{-18}$	(91.1876 ± 0.002) GeV/c^2
m_H / m_P	Higgs boson mass	$\approx 1.02 \times 10^{-17}$	(125.09 ± 0.24) GeV/c^2

Cosmological constants

The cosmological constant, which can be thought of as the density of dark energy in the universe, is a fundamental constant in physical cosmology that has a dimensionless value of approximately 10^{-122}. Other dimensionless constants are the measure of homogeneity in the universe, denoted by "Q" which is explained below by Martin Rees, the baryon mass per photon, the cold dark matter mass per photon and the neutrino mass per photon.[12]

Barrow and Tipler

Barrow and Tipler (1986) anchor their broad-ranging discussion of astrophysics, cosmology, quantum physics, teleology, and the anthropic principle in the fine structure constant, the proton-to-electron mass ratio (which they, along with Barrow (2002), call β), and the coupling constants for the strong force and gravitation.

Martin Rees's Six Numbers

Martin Rees, in his book *Just Six Numbers*, mulls over the following six dimensionless constants, whose values he deems fundamental to present-day physical theory and the known structure of the universe:

- $N \approx 10^{36}$: the ratio of the fine structure constant (the dimensionless coupling constant for electromagnetism) to the gravitational coupling constant, the latter defined using two protons. In Barrow and Tipler (1986) and elsewhere in Wikipedia, this ratio is denoted α/α_G. *N* governs the relative importance of gravity and electrostatic attraction/repulsion in explaining the properties of baryonic matter;[13]
- $\varepsilon \approx 0.007$: The fraction of the mass of four protons that is released as energy when fused into a helium nucleus. *ε* governs the energy output of stars, and is determined by the coupling constant for the strong force;[14]
- $\Omega \approx 0.3$: the ratio of the actual density of the universe to the critical (minimum) density required for the universe to eventually collapse under its gravity. Ω determines the ultimate fate of the universe. If $\Omega \geq 1$, the universe will experience a Big Crunch. If $\Omega < 1$, the universe will expand forever;[13]
- $\lambda \approx 0.7$: The ratio of the energy density of the universe, due to the cosmological constant, to the critical density of the universe. Others denote this ratio by Ω_Λ;[15]
- $Q \approx 10^{-5}$: The energy required to break up and disperse an instance of the largest known structures in the universe, namely a galactic cluster or supercluster, expressed as a fraction of the energy equivalent to the rest mass *m* of that structure, namely mc^2;[16]
- $D = 3$: the number of macroscopic spatial dimensions.

N and *ε* govern the fundamental interactions of physics. The other constants (*D* excepted) govern the size, age, and expansion of the universe. These five constants must be estimated empirically. *D*, on the other hand, is necessarily a nonzero natural number and cannot be measured. Hence most physicists would not deem it a dimensionless physical constant of the sort discussed in this entry.

Any plausible fundamental physical theory must be consistent with these six constants, and must either derive their values from the mathematics of the theory, or accept their values as empirical.

See also

- Cabibbo–Kobayashi–Maskawa matrix (Cabibbo angle)
- Coupling constant
- Dimensionless numbers in fluid mechanics
- Dimensionless quantity
- Dirac large numbers hypothesis
- Fine-structure constant
- Gravitational coupling constant
- Neutrino oscillation
- Physical cosmology
- Standard Model
- Weinberg angle
- Fine-tuned Universe
- Koide formula

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